L6 A Level Maths Assessment 2 Revision Questions Solutions

1.

$p = 0$ $q = -4$	1 1

2.

$-\frac{1}{2}$	**B1**

3.

$\frac{13}{6}$	**B1**

4.

b) $\frac{27^t}{3^{t-1}} = 3\sqrt{3}$

$\Rightarrow 27^t = 3^{t-1} \times 3\sqrt{3}$

$\Rightarrow (3^3)^t = 3^{t-1} \times 3^1 \times 3^{\frac{1}{2}}$

$\Rightarrow 3^{3t} = 3^{\boxed{t+\frac{1}{2}}}$

$\Rightarrow 3t = t + \frac{1}{2}$

$\Rightarrow 2t = \frac{1}{2}$

$\Rightarrow t = \frac{1}{4}$ //

5.

(a) $|AB| = \sqrt{(y_2-y_1)^2 - (x_2-x_1)^2} = \sqrt{(-1-3)^2 + (2-0)^2} = \sqrt{16+4} = \sqrt{20}$

(b) GRADIENT $AB = \frac{y_2-y_1}{x_2-x_1} = \frac{-1-3}{2-0} = \frac{-4}{2} = -2$

GRADIENT $BC = \frac{1-(-1)}{k-2} = \frac{2}{k-2}$

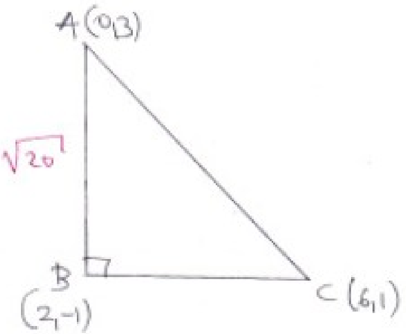
$\Rightarrow$ GRADIENTS ARE NEGATIVE RECIPROCALS

IF $\frac{2}{k-2} = \frac{1}{2}$

$\Rightarrow k-2 = 4$

$\Rightarrow k = 6$

(c)



$|BC| = \sqrt{(y_2-y_1)^2 + (x_2-x_1)^2}$

$= \sqrt{(1+1)^2 + (6-2)^2} = \sqrt{4+16}$

$= \sqrt{20}$

$\therefore$ AREA $= \frac{1}{2}|AB||BC|$

$= \frac{1}{2}\sqrt{20}\sqrt{20}$

$= 10$

6.

(a) $A(-6,4)$ & $B(3,16)$

GRADIENT $AB = \frac{y_2-y_1}{x_2-x_1} = \frac{16-4}{3+6} = \frac{12}{9} = \frac{4}{3}$

USING $A(-6,4)$: $y - y_0 = m(x - x_0)$

$\Rightarrow y - 4 = \frac{4}{3}(x+6)$

$\Rightarrow 3y - 12 = 4x + 24$

$\Rightarrow 3y = 4x + 36$

(b) $C(9,-1)$ & $D(-7,11)$

GRADIENT $CD = \frac{y_2-y_1}{x_2-x_1} = \frac{11+1}{-7-9} = \frac{12}{-16} = -\frac{3}{4}$

USING $C(9,-1)$: $y - y_0 = m(x - x_0)$

$\Rightarrow y + 1 = -\frac{3}{4}(x-9)$

$\Rightarrow 4y + 4 = -3x + 27$

$\Rightarrow 4y + 3x = 23$

(c) GRADIENT $L_1 = \frac{4}{3}$

GRADIENT $L_2 = -\frac{3}{4}$

GRADIENTS ARE NEGATIVE RECIPROCALS OF EACH OTHER, SO PERPENDICULAR

(d) $3y - 4x = 36 \quad (\times 3)$

$4y + 3x = 23 \quad (\times 4)$

$9y - 12x = 108$

$16y + 12y = 92$

ADD $25y = 200$

$y = 8$

& $3y - 4x = 36$

$\Rightarrow 24 - 4x = 36$

$\Rightarrow -4x = 12$

$\Rightarrow x = -3$

$\therefore E(-3,8)$

AS REQUIRED

7.

$x^2+4x+y^2-6y-8=0$ $(x+2)^2-4+(y-3)^2-9-8=0$ $(x+2)^2+(y-3)^2=21$ Centre (–2, 3) and radius $\sqrt{21}$	1 method 1 1

8.

$x^2+3x+k^2+2k-\frac{3}{4}=0$	1
If the line does not intersect the circle, then $b^2-4ac<0$ $3^2-4\times1\times\left(k^2+2k-\frac{3}{4}\right)<0$	1
$9-4k^2-8k+3<0$ $-4k^2-8k+12<0$	1
$4k^2+2k-3>0$ $(k-1)(k+3)>0$ $k<-3$ or $k>1$ Alternative method: Find the centre and radius of the circle and then consider which horizontal lines would intersect.	1

9.

a The centre of the circle $(x+4)^2+(y-1)^2=242$ is $(-4,1)$.
The gradient of the line joining (–4, 1) and (7, –10) is

$$\frac{y_2-y_1}{x_2-x_1}=\frac{-10-1}{7-(-4)}=\frac{-11}{7+4}=-\frac{11}{11}=-1$$

The gradient of the tangent is $\frac{-1}{(-1)}=1$.

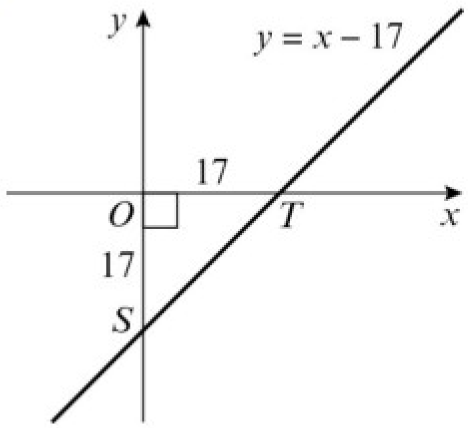
The equation of the tangent is

$$y-y_1=m(x-x_1)$$
$$y-(-10)=1(x-7)$$
$$y+10=x-7$$
$$y=x-17$$

Substitute $x=0$ into $y=x-17$
$y=0-17$
$y=-17$
So the coordinates of S are (0, –17)

Substitute $y=0$ into $y=x-17$
$0=x-17$
$x=17$
So the coordinates of T are (17, 0).

b



The area of ΔOST is $\frac{1}{2}\times17\times17=144.5$

10.

a Substitute $y = kx$ into $x^2 - 10x + y^2 - 12y + 57 = 0$

$$x^2 - 10x + (kx)^2 - 12kx + 57 = 0$$
$$(1 + k^2)x^2 - (10 + 12k)x + 57 = 0$$

a For two distinct points of intersection, $b^2 - 4ac > 0$

$$(-(10 + 12k))^2 - 4(1 + k^2)(57) > 0$$
$$144k^2 + 240k + 100 - 228k^2 - 228 > 0$$
$$-84k^2 + 240k - 128 > 0$$
$$21k^2 - 60k + 32 < 0$$

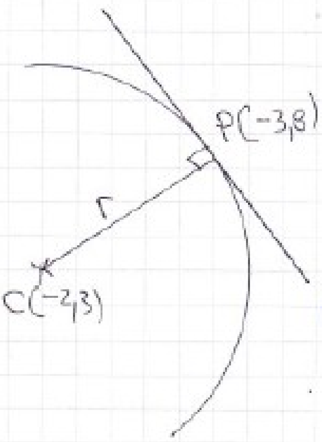
b Using the formula, $k = \dfrac{60 \pm \sqrt{(-60)^2 - 4(21)(32)}}{2(21)}$

$$k = \frac{60 \pm \sqrt{912}}{42}$$

$k = 0.71$ or $k = 2.15$,
$0.71 < k < 2.15$

11.

(a)



$$\text{RADIUS} = \sqrt{(8-3)^2 + (-3+2)^2}$$
$$= \sqrt{25+1} = \sqrt{26}$$

$$(x+2)^2 + (y-3)^2 = (\sqrt{26})^2$$
$$(x+2)^2 + (y-3)^2 = 26$$

(b) GRADIENT $PC = \dfrac{8-3}{-3+2} = \dfrac{5}{-1} = -5$

TANGENT GRADIENT IS $\frac{1}{5}$

$$y - y_0 = m(x - x_0)$$
$$y - 8 = \frac{1}{5}(x+3)$$
$$5y - 40 = x + 3$$
$$\therefore 0 = x - 5y + 43$$

12.

$y = x^5 - 6x^3 - 3x + 25$

when $x = 2$

$\Rightarrow y = 2^5 - 6 \times 2^3 - 3 \times 2 + 25$

$\Rightarrow y = 32 - 48 - 6 + 25$

$\Rightarrow y = 57 - 54$

$\Rightarrow y = 3$

$\therefore (2,3)$

$\frac{dy}{dx} = 5x^4 - 18x^2 - 3$

$\left.\frac{dy}{dx}\right|_{x=2} = 5 \times 2^4 - 18 \times 2^2 - 3$

$= 80 - 72 - 3$

$= 5$

$y - y_0 = m(x - x_0)$

$y - 3 = 5(x - 2)$

$y - 3 = 5x - 10$

$y = 5x - 7$

13.

(a) $y = \frac{6}{x^2} + \frac{5x}{4} - 4$

$y = 6x^{-2} + \frac{5}{4}x - 4$

$\frac{dy}{dx} = -12x^{-3} + \frac{5}{4}$

(OR) $\frac{dy}{dx} = \frac{5}{4} - \frac{12}{x^3}$

(b) when $x = 2$

$y = \frac{6}{2^2} + \frac{5\times2}{4} - 4$

$y = \frac{3}{2} + \frac{5}{2} - 4$

$y = 0$

$\therefore (2,0)$

$\left.\frac{dy}{dx}\right|_{x=2} = \frac{5}{4} - \frac{12}{2^3} = \frac{5}{4} - \frac{12}{8}$

$= \frac{5}{4} - \frac{6}{4} = -\frac{1}{4}$

$\therefore$ NORMAL GRADIENT IS 4, $(2,0)$

$\Rightarrow y - y_0 = m(x - x_0)$

$\Rightarrow y - 0 = 4(x-2)$

$\Rightarrow y = 4x - 8$

14.

(a) $f(x) = 4x\sqrt{x} - \frac{25x^2}{16}$

$\Rightarrow f(x) = 4x^1x^{\frac{1}{2}} - \frac{25}{16}x^2$

$\Rightarrow f(x) = 4x^{\frac{3}{2}} - \frac{25}{16}x^2$

$\Rightarrow f'(x) = 6x^{\frac{1}{2}} - \frac{25}{8}x$

(b) $f'(4) = 6\times4^{\frac{1}{2}} - \frac{25}{\cancel{8}_2}\times\cancel{4}$

$= 6\times2 - \frac{25}{2}$

$= 12 - \frac{25}{2}$

$= \frac{24}{2} - \frac{25}{2}$

$= -\frac{1}{2}$

when $x = 4$

$y = f(4) = 4\times4\times\sqrt{4} - \frac{25}{\cancel{16}}\times4^{\cancel{2}}$

$= 32 - 25$

$= 7$

$\therefore (4,7)$ & GRADIENT $-\frac{1}{2}$

$\Rightarrow y - y_0 = m(x - x_0)$

$\Rightarrow y - 7 = -\frac{1}{2}(x-4)$

$\Rightarrow 2y - 14 = -x + 4$

$\Rightarrow 2y + x = 18$

15.

(a) $f(x) = \frac{(2x-3)(x+2)}{\sqrt{x}} = \frac{2x^2+x-6}{x^{\frac{1}{2}}} = \frac{2x^2}{x^{\frac{1}{2}}} + \frac{x}{x^{\frac{1}{2}}} - \frac{6}{x^{\frac{1}{2}}}$

$= 2x^{\frac{3}{2}} + x^{\frac{1}{2}} - 6x^{-\frac{1}{2}}$

$A = 2$
$B = 1$
$C = -6$

(b) $f'(x) = 3x^{\frac{1}{2}} + \frac{1}{2}x^{-\frac{1}{2}} + 3x^{-\frac{3}{2}}$

$f'(1) = 3 + \frac{1}{2} + 3$

$f'(1) = \frac{13}{2}$

(IE GRADIENT OF TANGENT IS $\frac{13}{2}$)

ALSO $2y = 13x + 2$

$y = \frac{13}{2}x + 1$

SAME GRADIENT AS TANGENT

$\therefore$ INDEED PARALLEL

16.

(i) $f(x) = 2x^2 - 3x + 1$

$f(x+h) = 2(x+h)^2 - 3(x+h) + 1$

$= 2x^2 + 4xh + 2h^2 - 3x - 3h + 1$

Gradient of chord $= \frac{f(x+h) - f(x)}{(x+h) - x}$

$= \frac{2x^2 + 4xh + 2h^2 - 3x - 3h + 1 - (2x^2 - 3x + 1)}{x + h - x}$

$= \frac{4xh + 2h^2 - 3h}{h}$

$= 4x + 2h - 3$

As $h \to 0$, gradient of chord $\to 4x - 3$.

So $f'(x) = 4x - 3$.

(ii) $f(x)=x^3-2x^2+3$

$f(x+h)=(x+h)^3-2(x+h)^2+3$

$=x^3+3x^2h+3xh^2+h^3-2x^2-4xh-2h^2+3$

Gradient of chord $=\dfrac{f(x+h)-f(x)}{(x+h)-x}$

$=\dfrac{x^3+3x^2h+3xh^2+h^3-2x^2-4xh-2h^2+3-(x^3-2x^2+3)}{x+h-x}$

$=\dfrac{3x^2h+3xh^2+h^3-4xh-2h^2}{h}$

$=3x^2+3xh+h^2-4x-2h$

As $h\to 0$, gradient of chord $\to 3x^2-4x$.

So $f'(x)=3x^2-4x$.

17.

$f(x)=x^4$

$f(x+h)=(x+h)^4=x^4+4x^3h+6x^2h^2+4xh^3+h^4$

$f'(x)=\lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{(x+h)-x}$

$=\lim\limits_{h\to 0}\dfrac{x^4+4x^3h+6x^2h^2+4xh^3+h^4-x^4}{h}$

$=\lim\limits_{h\to 0}\dfrac{4x^3h+6x^2h^2+4xh^3+h^4}{h}$

$=\lim\limits_{h\to 0}\left(4x^3+6x^2h+4xh^2+h^3\right)$

$=4x^3$

18.

a $y=12x^2+3x+8$

$\dfrac{dy}{dx}=24x+3$

$\dfrac{d^2y}{dx^2}=24$

b $y=15x+6+\dfrac{3}{x}$

$=15x+6+3x^{-1}$

$\dfrac{dy}{dx}=15-3x^{-2}$

$\dfrac{d^2y}{dx^2}=6x^{-3}$

c $y=9\sqrt{x}-\dfrac{3}{x^2}$

$=9x^{\frac{1}{2}}-3x^{-2}$

$\dfrac{dy}{dx}=\dfrac{9}{2}x^{-\frac{1}{2}}+6x^{-3}$

$\dfrac{d^2y}{dx^2}=-\dfrac{9}{4}x^{-\frac{3}{2}}-18x^{-4}$

$\dfrac{d^2y}{dx^2}=-\dfrac{9}{4\left(\sqrt{x}\right)^3}-\dfrac{18}{x^4}$

d $y=(5x+4)(3x-2)$

$=15x^2+2x-8$

$\dfrac{dy}{dx}=30x+2$

$\dfrac{d^2y}{dx^2}=30$

e $y=\dfrac{3x+8}{x^2}$

$=\dfrac{3x}{x^2}+\dfrac{8}{x^2}$

$=\dfrac{3}{x}+8x^{-2}$

$=3x^{-1}+8x^{-2}$

$\dfrac{dy}{dx}=-3x^{-2}-16x^{-3}$

$\dfrac{d^2y}{dx^2}=6x^{-3}+48x^{-4}$